00 P2 A6	**6A** •1 $A'(x) = \ldots\ldots$ •2 $\frac{3\sqrt{3}}{2}\left(2x - 16x^{-2}\right)$ *or* $3\sqrt{3}x - 24\sqrt{3}x^{-2}$ •3 $A'(x) = 0$ •4 $-\frac{16}{x^2}$ *or* $-\frac{24\sqrt{3}}{x^2}$ •5 $x = 2$ •6 x 2^- 2 2^+ $A'(x)$ $-ve$ 0 $+ve$ so $x = 2$ is min.
01 P1 Q6	**5C** •1 $\frac{dP}{dx} = 36x^2 \ldots\ldots$ **or** $\frac{dP}{dx} = \ldots\ldots - 4x^3$ •2 $\frac{dP}{dx} = 36x^2 - 4x^3$ •3 $\frac{dP}{dx} = 0$ •4 $x = 0$ ***and*** $x = 9$ •5 nature table about $x = 9$ *and* $x = 9$
02 P2 Q10	**3A, 4B** •1 proof of $l = \frac{5}{4}a$ •2 $b = \frac{3}{5}(8 - a)$ •3 complete proof leading to $A = \ldots$ •4 $\frac{dA}{da} = \ldots 0$ •5 $6 - \frac{3}{2}a$ •6 $a = 4$ •7 *e.g.* nature table, comp the square
3. JAN) 02 P2	**5B** •1 $\frac{dS}{dw} = \ldots = 0$ stated explicitly •2 $680 - 5.1w^2$ •3 $w = \frac{20}{\sqrt{3}}$ (11.5) •4 *e.g.* nature table •5 $d = 20\sqrt{\frac{2}{3}}$ (16.3)
03 P2 Q8	**3A, 5B**

	$\bullet^1$ $length = \frac{108000}{\frac{1}{2}x^2}$ $\bullet^2$ $SA = 2\times\frac{1}{2}x^2 + 2x\times length$ $\bullet^3$ $\ldots SA = x^2 + \frac{432000}{x}$ $\bullet^4$ $\frac{dA}{dx} = \ldots = 0$ $\bullet^5$ $432000x^{-1}$ $\bullet^6$ $2x - 432000x^{-2}$ $\bullet^7$ $x = 60$ $\bullet^8$ $e.g.$ nature table $\begin{array}{c\|ccc\|} x & 60^- & 60 & 60^+ \\ \frac{dA}{dx} & -ve & 0 & +ve \\ & \backslash & — & / \end{array}$ minimum
04 P2 Q9	**3A/B, 5C** $\bullet^1$ $A = 2x^2 + 2xh + 4xh = 12$ $\bullet^2$ $V = 2x \times x \times h$ $\bullet^3$ $V = 2x \times \frac{12-2x^2}{6} =$ & $complete$ $\bullet^4$ $V = 4x - \frac{2}{3}x^3$ $\bullet^5$ $\frac{dV}{dx} = 4 - 2x^2$ $\bullet^6$ $\frac{dV}{dx} = 0$ *STATED EXPLICITLY* $\bullet^7$ $x = \sqrt{2}$ $\bullet^8$ $\begin{array}{c\|ccc} x & <\sqrt{2} & \sqrt{2} & >\sqrt{2} \\ \hline \frac{dV}{dx} & +ve & 0 & -ve \\ \hline tgt & / & - & \backslash \end{array}$ max **OR** $\bullet^1$ $2x^2 + 2xh + 4xh = 12$ $\bullet^2$ $h = \frac{12-2x^2}{6x}$ $\bullet^3$ $V = 2x \times x \times \frac{12-2x^2}{6x} =$ & $complete$
06 P2 Q12	**3A, 8A/B**

<table>
<tr>
<td></td>
<td>
•1 $PS = 6 - x$
•2 $RS = 12 - \dfrac{8}{x}$
•3 $Area = (6 - x)\left(12 - \dfrac{8}{x}\right)$ and complete
•4 $48x^{-1}$
•5 $\dfrac{dA}{dx} = 0$
•6 $-12 + 48x^{-2}$
•7 $x = 2$
•8 $A(2) = 32$
•9 $A(1) = 20$
•10 $A(4) = 20$
•11 $\max.A = 32 \ at \ x = 2 \ \textbf{and}$
$\min.A = 20 \ at \ x = 1 \ or \ x = 4$
</td>
</tr>
<tr>
<td>07 P2
Q6</td>
<td>
3A, 5C/B
•1 $ST = \sqrt{200}$
•2 $length = \sqrt{200} - 2x$ s/i by their method
•3 $\left(\sqrt{200} - 2x\right) \times x$
$and \ complete \ proof$
•4 $\dfrac{dA}{dx} = 0$
•5 $\dfrac{dA}{dx} = 10\sqrt{2} - 4x$
•6 $x = \dfrac{10\sqrt{2}}{4}$ or equivalent (3.5)
•7 $justification$: e.g. nature table
•8 $length = 5\sqrt{2}$ (7.1)
Minimum requirement of a nature table
<table>
<tr><td></td><td>...</td><td>3.5</td><td>...</td></tr>
<tr><td>$f'(x)$</td><td>$+$</td><td>0</td><td>$-$</td></tr>
</table>
hence maximum

better would be
<table>
<tr><td>x</td><td>$\rightarrow$</td><td>$\frac{5\sqrt{2}}{2}$</td><td>$\rightarrow$</td></tr>
<tr><td>$f'(x)$</td><td>$+$</td><td>0</td><td>$-$</td></tr>
<tr><td>$f(x)$</td><td>$\therefore$</td><td>$\cdots$</td><td>$\ddots$</td></tr>
</table>
hence maximum
at $x = \frac{5\sqrt{2}}{2}$
</td>
</tr>
<tr>
<td>08 P2
Q6</td>
<td>3A, 6B</td>
</tr>
</table>

	•[1] ΔOST, RSQ are similar s/i by •[2] •[2] $\dfrac{QR}{6} = \dfrac{3-t}{3}$ or equivalent •[3] $QR = 6 - 2t$ •[4] $A(t) = t(6-2t)$ •[5] $A'(t) = 0$ •[6] $6 - 4t$ •[7] $t = \frac{3}{2}$ •[8] *e.g. nature table* •[9] $Q = \left(\frac{3}{2}, 3\right)$	**Nature Table** **minimum requirements for •[8]** •[8] $\begin{array}{c\|ccc\|} & & \frac{3}{2} & \\ A' & + & 0 & - \\ & \nearrow & \dots & \searrow \end{array}$
10 P2 Q5	know to and find OT obtain an expression for PQ complete area evaluation	•[1] 4 or $(0,4)$ •[2] $10 - x^2 - 4$ •[3] $2x \times (6 - x^2) = 12x - 2x^3$
	know to and start to differentiate complete differentiation set derivative to zero obtain x	•[4] $A'(x) = 12 \dots$ *stated* •[5] $12 - 6x^2$ •[6] $12 - 6x^2 = 0$ •[7] $\sqrt{2}$ or decimal equivalent
	justify nature of stationary point	•[8] $\begin{array}{c\|ccc} x & \dots & \sqrt{2} & \dots \\ \hline A'(x) & + & 0 & - \end{array}$
	interpret result and evaluate area	•[9] Max **and** $8\sqrt{2}$ or decimal equivalent
13 P2 Q7	identify expression for L in x and y identify expression for y in terms of x complete proof	•[1] $L = 3x + 4y$ •[2] $y = \dfrac{24}{2x}$ •[3] $L = 3x + 4 \times \dfrac{24}{2x}$ and complete
	prepare to differentiate differentiate equate derivative to 0 process for x verify nature evaluate L evaluate cost	•[4] $\dots 48x^{-1}$ •[5] $3 - 48x^{-2}$ •[6] $3 - 48x^{-2} = 0$ •[7] $x = 4$ •[8] nature table or 2^{nd} derivative •[9] $L = 24$ •[10] cost $24 \times £8{\cdot}25 = £198$

	$$\begin{array}{c|ccc} x & \longrightarrow & 4 & \longrightarrow \\ \hline \frac{dL}{dx} & - & 0 & + \\ & & \text{Min} & \end{array}$$

ANSWERS **Pre 2000 - Optimisation**

1	(a) •1 $9y+8x$ •2 $A=3y\times 2x$ •3 $9y=(360-8x)$ •4 $2x.3.\frac{1}{9}(360-8x)$ and complete proof	(b) •5 $A'(x)=\ldots\ldots$ •6 $240-\frac{32}{3}x$ •7 $A'(x)=0$ *or* $240-\frac{32}{3}x=0$ •8 $x=22\frac{1}{2},\ \ y=20$ •9 $\begin{array}{c\|ccc} x & 22\frac{1}{2}^- & 22\frac{1}{2} & 22\frac{1}{2}^+ \\ \hline A'(x) & + & 0 & - \\ & & \text{maximum} & \end{array}$ •10 2700
2	(a) •1 introduce height specific to this cuboid •2 $h=\frac{500}{x^2}$ •3 $A=x^2+4xh$ •4 $A=x^2+4x.\frac{500}{x^2}$ explicitly stated	(b) •5 $A'(x)=\ldots\ldots$ •6 $2x-2000x^{-2}$ •7 $A'(x)=0$ specifically stated •8 $x=10$ •9 justify minimum e.g. with table •10 dimensions of 10 by 10 by 5
3	(a) •1 rectangle minus 3 triangles •2 area of Δ's ADF and ABE •3 area of Δ FCE •4 3 triangles : $24+4x-\frac{1}{2}x^2$ or $48-4x-3x+\frac{1}{2}x^2-24+3x$ (b) •5 $H'(x)=\ldots\ldots$ •6 $x-4$ •7 put $H'(x)=0$ **stated explicitly** •8 $x=4$ and $H=16$ •9 justify minimum •10 consider $x=0$ **and** $x=6$ •11 $H(0)=24$, **and** $H(6)=18$ •12 communication re greatest and least.	

4

(a)

•1 strategy: e.g. equate ratios from similar triangles

•2 $\frac{10}{4} = \frac{10-h}{x}$ or equivalent

•3 complete proof

•4 $V = 40x^2 - 10x^3$

(b)

•5 $\frac{dV}{dx} =$

•6 $80x - 30x^2$

•7 $\frac{dV}{dx} = 0$ for stationary points

•8 $0, \frac{8}{3}$

•9

x	…	$\frac{8}{3}$	…
$\frac{dV}{dx}$	+	0	−
		max	

•10 $\frac{16}{3}$ and $\frac{10}{3}$

5

(a)

•1 $\pi r^2 + 2\pi rh + 2\pi r^2$

•2 $h = \frac{400}{\pi r^2}$ or equivalent (*e.g.* $\pi rh = \frac{400}{r}$)

•3 $2\pi r \frac{400}{\pi r^2} + 3\pi r^2$ and completes proof

(b)

•4 $\frac{dA}{dr} = \ldots$

•5 $800r^{-1}$

•6 $6\pi r - 800r^{-2}$

•7 *e.g.* $6\pi r - \frac{800}{r^2} = 0$

•8 3.5

•9

r	3.5^-	3.5	3.5^+
$\frac{dA}{dr}$	$-ve$	0	$+ve$

6

(a)

•1 eg $2h + 2x + \text{semicircle} = 10$

•2 $h = \frac{1}{2}(10 - \pi x - 2x)$

•3 $L = 2 \times 2xh + \frac{1}{2}\pi x^2$

•4 $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$

$L = 20x - 4x^2 - \frac{3}{2}\pi x^2$

(b)

•5 $L' = 20 - 8x - 3\pi x$

•6 $L' = 0$

•7 $x = \frac{20}{3\pi + 8} = x_0$ $(= 1.148)$

•8

x	x_0^-	x_0	x_0^+
L'	+	0	−
maximum at x_0			

•9 $h = \frac{5\pi + 20}{3\pi + 8}$ $(= 2.049)$

7	**(a)** •1 $C = 2x + y$ •2 $\sqrt{x^2 - \left(9\sqrt{3}\right)^2}$ •3 for completing proof **(b)** •4 knowing to differentiate •5 $\frac{1}{2}\left(x^2 - 243\right)^{-\frac{1}{2}}$ •6 $\times\ 2x$ •7 $C'(18) = 0$ •8 justification of minimum e.g. nature table •9 $C = 127$ •10 $x + y = 109$
8	**(a)** •1 $B = (x, y)$ where $y = 9 - \frac{1}{4}x^2$ •2 area $= 2x\left(9 - \frac{1}{4}x^2\right)$ **(b)** •3 $V = 1080x - 30x^3$ •4 $\frac{dV}{dx} = 1080 - 90x^2$ •5 $\frac{dV}{dx} = 0$ **stated explicitly** •6 $x = 2\sqrt{3}$ •7 x $2\sqrt{3}^-$ $2\sqrt{3}$ $2\sqrt{3}^+$ $\frac{dV}{dx}$ $+$ 0 $-$ •8 max at $x = 2\sqrt{3}$ of $1440\sqrt{3}$
9	**(a)** •1 $T = x + x + y$ **and** $y^2 = 20 - x^2$ **(b)** •2 appearance of $\frac{dT}{dx} = 2 + \ldots\ldots$ •3 $\frac{1}{2}\left(20 - x^2\right)^{-\frac{1}{2}}$ •4 $\times -2x$ •5 $\frac{dT}{dx} = 0$ stated or implied •6 completing proof **(c)** •7 $x^2 = 4\left(20 - x^2\right)$ •8 $x = 4$ (accept $x = \pm 4$) •9 justifying $x = 4$ gives $T_{max} = 10$

Nature table for 7(b):

	18^-	18	18^+
$C'(x)$	$-$	0	$+$
	$\searrow$	$\rightarrow$	$\nearrow$
		minimum	